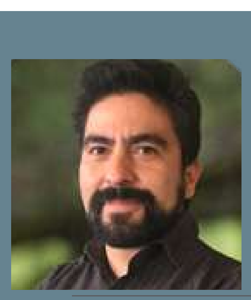
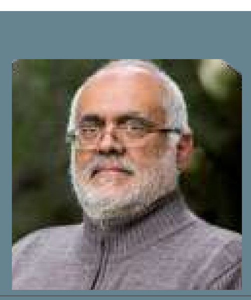
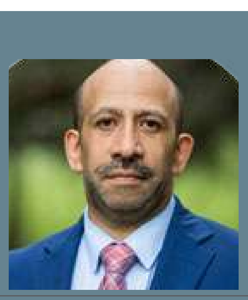
THE TABLES OF THE (NORMAL) LAW

Anáhuac México


José Daniel López-Barrientos
Facultad de Ciencias Actuariales
daniel.lopez@anahuac.mx


Enrique Lemus-Rodríguez
Facultad de Ciencias Actuariales
elemus@anahuac.mx


José Eliud Silva-Urrutia
Facultad de Ciencias Actuariales
jose.silva@anahuac.mx

INTRODUCCIÓN

In this work, we intend to give the students a hands-on approach to one of the most important topics in the first course of univariate families of parametric probability distributions: the Central Limit Theorem (CLT). To this end, we have changed the traditional orders in which the topics are presented [for example, see 1,2] and proposed a set of computational, mathematical, and practical activities that enable the students to understand both: the roots of the result, and its deep connection with the rest of the topics in Probability Theory, Risk Theory, and Data Science. One of our main contributions is the fact that the experiences that we propose are *organic*, in the sense that they require the students to input the data, codes, and formulae by themselves, rather than limiting them to the observation of a pre-programmed applet to generate knowledge [as in 3]. In this context, it is fundamental to acknowledge that the *sketch of the proof* of the CLT presented in Chapter 10.3 [4] is very accurate, and even quotes the complete proof [5, p. 233].

METODOLOGÍA

We re-order some of the topics from the traditional ordering used in a Probability course, and include some others from other disciplines in a sequence related to ten commandments: 1. Thou shalt read the introduction, 2. Thou shalt not scam your neighbor, 3. Thou shalt share thy birthday, 4. Thou shalt use trapezoids to approximate logarithm, 5. Lisez Euler, lisez Euler, c'est notre maître à tous, 6. Thou shalt play matching pennies, 7. Thou shalt plant a (binomial) tree, 8. Thou shalt record thy goals, 9. Honour De Moivre and Laplace, and 10. Thou shalt not doubt the strength of thy Theorem

RESULTADOS

- A heuristic on the CLT through an arranged board game where one faction will scam the other by means of a misuse of the theorem. This activity includes the input of data in a spreadsheet and coding the convolution formula in VBA.
- An activity where we use the birthday paradox so that the members of the control group get insights of the multiplication rule. We also recur to the logarithmic function to simplify the calculations and to approximate the probability of avoiding collisions in k trials in the birthday paradox context as

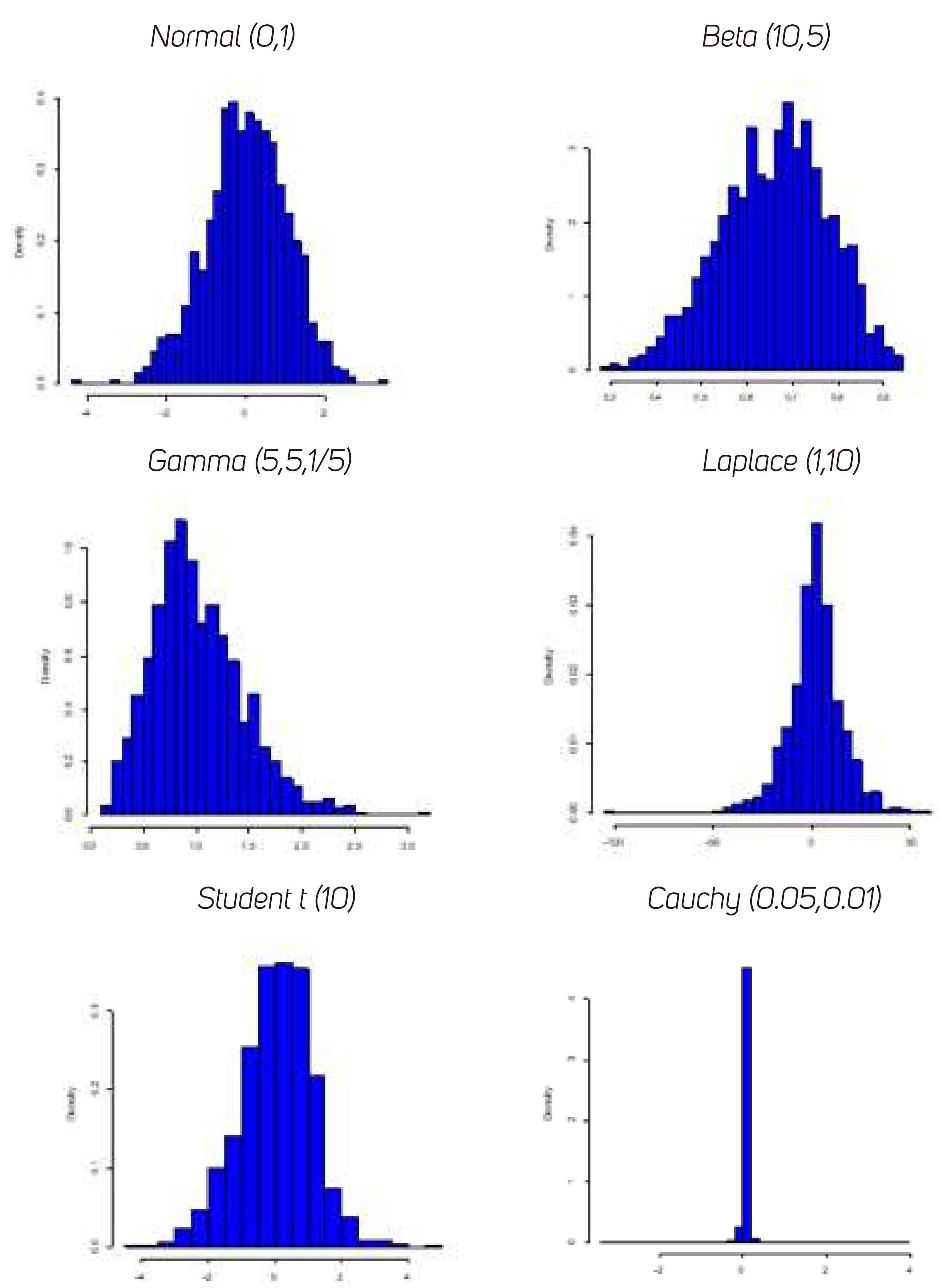
$$e^{-\frac{1}{2n}(k^2-k)}$$

where n stands for the number of days in the period under consideration.

A variant of the "Matching Pennies" game [6, p. 17] to introduce Bernoulli random variables, both Laws of Large Numbers; and the Monte Carlo simulation technique. In particular we defined

$$\binom{n}{k} := \frac{d^k}{dx^k}\frac{(1+x)^n}{k!}$$

- A modification of the approach presented [7] to introduce the Poisson process and the Exponential distribution.
- A sketch of the proof of the CLT based on Chapter 5 [8], and a set of activities in the statistical language R to test the hypotheses of the CLT, and use it in several random variables, which are usual in the actuarial context. We used several normality tests, among which we can find Jarque-Bera's, due to a former student of the Facultad de Ciencias Actuariales, Carlos Jarque-Uribe (Act. 1976), and Anil K. Bera [9].



DISCUSIÓN

The sketch of the proof that we present to the students delves with some of the finer points of the actual proof, which otherwise are frivolously studied, or simply omitted in other references. For example, in the "proof" presented by Theorem 5.10.4 [10], there is an implicit use of the fact that, if the limiting characteristic function of a sum of random variables coincides with the characteristic function of another random variable, then the underlying density functions are the same. We have taken advantage of the aim of our study to survey a broad range of the most important topics in a first course of univariate families of parametric probability distributions, and we have managed to leave meaningful insights to the students in order to help them take on the subject by themselves.

REFERENCIAS

1. Billingsley P. Convergence of Probability Measures, John Wiley & Sons, Inc; 2013.
2. Hoel P, Port S, Stone C. Introduction to Probability Theory, Boston: Houghton Mifflin Company; 1971.
3. Dinov ID, Christou N, Sanchez J. Central Limit Theorem: New SOCR Applet and Demonstration Activity, Journal of Statistics Education, 2008;16(2).
4. Grinstead CM, Snell JL. Introduction to Probability, American Mathematical Society; 1997.
5. Gnedenko BV, Kolmogorov AN. Limit Distributions for Sums of Independent Random. Addison-Wesley; 1968.
6. Haurie A, Krawczyk JB, Zaccour G- Games and Dynamic Games, World Scientific Publishing; 2012.
7. Sumpter D. Soccermatics: Mathematical Adventures in the Beautiful Game Pro-Edition, Bloomsbury Sigma; 2017.
8. Ball K, Strange Curves, Counting Rabbits, and other Mathematical Explorations, London: Princeton University Press; 2003.
9. Jarque-Uribe CM, Bera AK. A test for normality of observations and regression residuals, International Statistical Review, 1987;55(2):163-172.
10. Grimmett G, Stirzaker D. Probability and random processes, Oxford Science Publications; 1994.